1617-257 Fri Sep 16, Hour 3: Topology in R^n

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Riddle. Can you find uncountably many disjoint open subsets of R? Of R^n? Closed instead of open?

on board

Today: Topology in $\mathbb{R}^n$.

Read Along: Munkres sec 3

Metric on a set X:

1. Symmetric 2. positivity 3. triangle.

Metric space, subspace

Examples: 1. $\mathbb{R}^n, \|\ \|$, $\mathbb{R}^n, |\ |$

2. Discrete

3. $C([0,1])$, sup norm.

$U(x_0,\epsilon)$ "ϵ-nbd", "ϵ-ball".

Open set, closed set.

$U(x_0,\epsilon)$ is open

Thm 1a 1. $\emptyset, X$ are open

2. V_α open $\Rightarrow \bigcup V_\alpha$ is open

3. V_i open $i=1,\dots n \Rightarrow \bigcap_{i=1}^n V_i$ is open

no proofs done line

Thm 4a $U \subset \mathbb{R}^n$ is $\|\cdot\|$-open iff it is $|\cdot|$-open.

Thm 1b 1. $\emptyset, X$ are closed

2. F_α closed $\Rightarrow \bigcap F_\alpha$ is closed.

3. F_i closed, $i=1\dots n \Rightarrow \bigcup F_i$ is closed.

Thm 4b $F \subset \mathbb{R}^n$ is $\|\cdot\|$-closed iff it is $|\cdot|$-closed.

Thm 2 $Y \subset X$ a subspace of a metric space.

Then $A \subset Y$ is open iff $\exists U \subset X$ open s.t. $A = Y \cap U$, and $B \subset Y$ is closed iff $\exists F \subset X$ closed s.t. $B = Y \cap F$.

Def Limit x_0 of $A \subset X$:

$$\forall \epsilon > 0 \quad \left(U(x_0, \epsilon) \setminus \{x_0\} \right) \cap A \neq \emptyset$$

Equiv: every nbd of x_0 contains ∞-many elements of A.

Closure: $\overline{A} = A \cup \{\text{limit pts of } A\}$

Thm A is closed $\iff A = \overline{A}$

Suppose X & Y are metric, w/ metrics d_X & d_Y

Def $f: X \to Y$ is cont. at $x_0 \in X$ if for every nbd V of $f(x_0)$ [:= an open set containing $f(x_0)$] the is a nbd U of x_0 s.t. $f(U) \subset V$

$\iff$

$$\forall \epsilon > 0 \; \exists \delta > 0 \quad d_X(x, x_0) < \delta \Rightarrow d_Y(f(x), f(x_0)) < \epsilon$$

Def $f: X \to Y$ is cont. means $\forall x_0 \in X$, f is cont. at x_0.

Thm TFAE for $f: X \to Y$:

1. f is continuous.
2. For every V open in Y, $f^{-1}(V)$ is open in X
3. For every F closed in Y, $f^{-1}(F)$ is closed in X
4. if $X = Y = \mathbb{R}$, f is cont. in the 157 sense.

target line

Thm 1. constant functions are continuous.

2. $I: X \to X$ is cont.

3. $f: X \to Y$ cont, $A \subset X \Rightarrow f|_A: A \to Y$ is cont.

4. $f: X \to Y$, $g: Y \to Z \Rightarrow f /\!/ g = g \circ f$ is cont.

5. $f: X \to \mathbb{R}^n$ is $(f_1(x), f_2(x), \ldots f_n(x))$.

Then f is cont $\iff \forall i \; f_i$ is cont.

6. $f, g: X \to \mathbb{R}$ cont $\Rightarrow f+g,\ f \cdot g,\ f-g,\ \frac{f}{g}$ (where defined)

cont.

7. $\pi_i : \mathbb{R}^n \to \mathbb{R}$ is cont.

HW: Read the rest of section 3, about limits, interiors, exteriors.